Specialized Topics

1 Introduction

The "Specialized Topics" questions on the Physics GRE are probably the most unique aspect of the test. It's hard to think of any other test, standardized or otherwise, in which a full 10% is random assorted knowledge. This may seem daunting, but with smart preparation, these questions actually offer a huge advantage.

The special topics questions are almost entirely pure knowledge recall, otherwise known as fact regurgitation. This is the kind of knee-jerk memorization you probably hated in high-school chemistry. When confronted by a special topics question, you'll either know it or you won't. If you know it, that's one question down in under 10 seconds, which gives you a *huge* bonus on time for the more difficult calculational questions. If you don't know it, you probably won't be able to figure out the answer just by reasoning through it, and you may waste 5 or more minutes second-guessing yourself when stuck between two equally appealing answer choices. The optimum strategy, then, is to amass a basic knowledge of as many areas of cutting-edge physics as possible, *just enough* to make the associations between "buzzwords" and concepts that will allow you to recall the required knowledge.

Luckily, this kind of studying is *dead easy*. Every couple days, take a break from your normal Physics GRE practice and just *read*. Pick up a basic textbook in an advanced subject you're unfamiliar with (for example, if you're aiming towards high-energy, choose an introductory solid-state physics or electrical engineering textbook), and don't bother working any problems; just read the book as if it were a novel. You might learn something new and interesting, but that's not really the point: by reading this way, you'll be forming connections and associations in your memory that you might not even be aware of. It's likely you won't be able to remember exactly what you read, but if prompted by a keyword that shows up on the GRE, your memory will spring into action with that feeling of "I've seen this somewhere before." That's really all you need for these kinds of questions.

To give you a headstart, we've collected here some of the material which is most likely going to be tested in these kinds of questions. The tone of this section will be much more informal than the rest of the book, and some concepts are purposefully not explained in gory detail: as mentioned above, you won't need to know these kinds of details, so consider this leisure reading. When you're sick of doing problems, revisit this section and read a few paragraphs.

As for more detailed reference material, a *mandatory* read is the first chapter of *Introduction to Elementary Particles* by Griffiths. It seems that every GRE in the last three years has contained at least one question which can be answered purely by picking facts out of this chapter.

2 Nuclear and particle physics

2.1 The Standard Model: particles and interactions

The modern description of the particles and forces found in nature is a relativistic quantum field theory called the Standard Model. The spin-1 *bosons* of the Standard Model mediate the fundamental forces: photons mediate the electromagnetic force, $W^\pm$ and Z bosons mediate the weak nuclear force, and gluons mediate the strong nuclear force. (There are also hypothetical spin-2 gravitons, which mediate the gravitational force, but despite our best efforts these have not been experimentally observed.) The photon and gluons are massless, whereas the W and Z bosons are extremely heavy, about 90 times the mass of the proton. For group-theoretical reasons, there are eight gluons. The photon and the Z are their own antiparticles, whereas the W^+ is the antiparticle of the W^-.

The spin-1/2 *fermions* of the standard model, collectively known as *matter*, can be organized in three *generations*. In each generation, there are two *quarks*, one electron-type particle, and one *neutrino*. The first generation contains the up quark, down quark, electron, and electron neutrino; the second contains the charm quark, strange quark, muon, and mu neutrino; and the third contains the top quark, bottom quark, tau, and tau neutrino. Each generation is successively heavier than the next, with the muon more than 200 times the mass of the electron, and the tau about 1000 times the mass of the muon. Because of the large mass hierarchy between generations, third- and second-generation particles will tend to decay to first-generation particles. Indeed, *all* the stable matter in the universe consists of first-generation particles (plus all flavors of neutrinos – more on this later).

The quarks interact via the strong nuclear force, but they also carry electric charge: up-type quarks have charge $+2/3$, and down-type have $-1/3$, in units of the magnitude of the electron charge. In fact, they interact via the weak nuclear force as well: emitting or absorbing a W-boson can change the *flavor* of a quark, from an up-type to a down-type or vice-versa. The electron-type particles and neutrinos, collectively known as *leptons*, interact via the electromagnetic and weak forces, but *not* the strong force. The electron, muon, and tau all have charge -1, but the neutrinos are electrically neutral. Finally, because of relativity, each particle has an antiparticle, so there are anti-particle partners for each matter particle. The anti-electron is more commonly known as the positron, but antiparticles of the other quarks and leptons are usually either given the prefix "anti-" or labeled by their opposite charge (μ^+, pronounced "mu-plus," for the anti-muon).

The forces in the Standard Model are of a very special type known as *gauge theories*, which very roughly means they result from a symmetry which can act independently at every point in space. This symmetry is implemented by a mathematical *group*: U(1) for electromagnetism, SU(2) for the weak interaction, and SU(3) for the strong interaction, also known as *quantum chromodynamics* or *QCD*. Actually, this is a bit of a fudge: at high energies, the electromagnetic and weak interactions unify into a single force with group SU(2), and the U(1) described above is really a scaled-down version of electromagnetism called *hypercharge*. The SU(3) symmetry is called *color*, and we say that quarks come in three colors, so each generation really contains six quarks.

2.2 Nuclear physics: bound states

All ordinary matter in the universe is protons, neutrons, and electrons. We've already addressed electrons above: these are elementary particles. However, protons and neutrons are *composite* – in the framework of quantum field theory, they are teeming seas of quarks and gluons constantly popping in and out of existence. At low energies, where nuclear physics is applicable, we can simplify this description considerably using the *quark model*. Here, the proton is considered a bound state of two up quarks and a down quark, for a total charge of $2(2/3) - 1/3 = +1$, and the neutron is a bound state of two down quarks and an up quark, for a total charge of $2(-1/3) + 2/3 = 0$. Due to a property of QCD called *confinement*, free quarks cannot be seen in nature, and instead they collect themselves into bound states like protons and neutrons. All of these bound states are color-neutral, also referred to as *color singlets*.

If we collide strongly-interacting particles together at higher and higher energies, we can form all kinds of different bound states, heavier than the proton and neutron. These fall into two general categories: *mesons*, made of a quark and an antiquark, and *baryons*, made of three quarks or three anti-quarks. Note that because of the rules for adding angular momentum, mesons may have either spin-1 or spin-0, whereas baryons may have spin-3/2 or spin-1/2. In the 1960's, it was observed that mesons and baryons made out of only the three lightest quarks (up, down, and strange) arranged themselves into interesting patterns which Gell-Mann called *the Eightfold Way*. Historically, mesons and baryons were the vehicles by which new generations of quarks were discovered: see Griffiths for more information.

The lightest mesons are called the *pions*, and there are three of them: the charged pions $\pi^{\pm}$ and the neutral pion π^0. Because they are so light, they are copiously produced in any strongly interacting process, including the bombardment of our atmosphere by cosmic rays, which is how they were first discovered. Their decay modes are also useful to remember: the charged pions decay principally into muons and mu-neutrinos, while the neutral pion decays principally into *two* photons. (Why not one photon? Go to the rest frame of the pion: decay to a single massless photon is forbidden by relativistic conservation of energy/momentum.) Because of confinement, there are no free gluons at low energies, so it is actually the pion which mediates the strong nuclear force between protons and neutrons. The fact that the pion is massive, rather than massless, results in an exponential suppression of the strength of the force at distances larger than the Compton wavelength of the pion, accounting for the short range of this force. Incidentally, because the pions are spin-0, the strong force at low energies is *universally attractive*: as far as protons and neutrons are concerned, there is no such thing as positive or negative "strong charge."

The lightest baryons are the proton and the neutron: the fact that the neutron is (very) slightly heavier than the proton means that the neutron can decay via $n \to p + e^- + \bar{\nu}_e$. Indeed, free neutrons *do* decay, with a lifetime on the order of 15 minutes, but inside a nucleus the constant strong interactions with protons keep them from decaying immediately. In certain nuclei, though, the neutron can decay via quantum tunneling, in a process better known as *beta decay.* As the nuclei get bigger and bigger, the electromagnetic repulsion between protons starts to cancel the attractive effects of the strong force, and whole chunks of the nucleus can break off: this leads to *alpha decay*, emission of bound states of two protons and two neutrons (in other words, helium-4 nuclei). The final type of radiation, *gamma* radiation, is the emission of photons from an excited state of a nucleus, which doesn't change the proton/neutron composition of the nucleus. Such excited states may actually be interpreted as different particles: for example, the first excited state of the proton is known as the Δ^+, which has the same quark content as the proton but a larger mass. If these particles are created or observed outside the nucleus, they are considered baryons in their own right.

In addition to processes where nuclei can break apart (*fission*), sufficiently small nuclei can also join together through *fusion.* This requires enormous temperatures and pressures in order to overcome the electromagnetic repulsion between the protons, but can also release enormous amounts of energy; indeed, both the sun and the hydrogen bomb are powered by fusion reactions. In the sun, successive protons are fused onto larger and larger nuclei, with some being converted to neutrons along the way, all the way up to ${}^4_2\mathrm{He}$. In the standard picture of the genesis of heavy elements in the early universe, light nuclei continued to fuse as a result of supernovas, all the way up to iron (atomic number 26), which is the most stable nucleus. Heavier nuclei become progressively more and more unstable, up to lead (atomic number 82), beyond which all heavier nuclei will eventually decay.

2.3 Symmetries and conservation laws

The general rule of particle physics is that anything that *can* happen, *will* happen, unless it is forbidden by a symmetry or a conservation law. For example, the electron is the lightest negatively charged particle (excluding quarks, which as we've discussed can't exist free in nature), so it is forbidden from decaying by conservation of charge. But the proton is heavier than the positron – why doesn't it decay? To explain this, and other similar observations, a new law called *conservation of baryon number* was introduced. Baryons get baryon number $+1$, anti-baryons get -1, and everything else is assigned zero. Similarly, the fact that an extra neutrino is *always* produced in beta decay suggests *conservation of lepton number*, which is really three separate conservation laws (conservation of electron number, muon number, and tau number), where in each generation the lepton and its associated neutrino are given lepton number $+1$ and corresponding antiparticles get -1. Baryon and lepton number are known as *continuous symmetries*, essentially because they correspond to a conserved *charge*, namely total baryon or lepton number.

For example, we can apply these rules to find the possible decay modes of the muon μ^-. By conservation of mu number, we must have a mu neutrino in the decay products.

By conservation of charge, we must have a lighter negatively charged particle: the only stable option is the electron. But by conservation of electron number, we must have an accompanying neutral particle with electron number −1, namely the anti-electron-neutrino. This fixes one decay mode to be

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e.$$

Other decays are possible, but they must all contain pairs of particles with net charge and lepton number zero: for example, a pair of photons, or an electron-positron pair.

There are also various *discrete* symmetries which are important in nature. The symmetry operation P, or *parity*, reverses the orientation of space, which is another way of saying P takes a configuration of charges to its mirror image. The symmetry operation C, *charge conjugation*, exchanges particles and antiparticles. Finally, T, or *time-reversal*, does what it sounds like. A very important theorem in quantum field theory states that all Lorentz-invariant local quantum field theories must be symmetric under the combined action of all these operations, known as CPT. However, it is an important and striking fact that the Standard Model does *not* respect each of these symmetries individually. The weak interaction is said to be *maximally parity-violating*: the classic experiment which proved this looked at beta-decay of cobalt-60, and found that the decay products were preferentially produced in one direction relative to the spin of the nucleus. In fact, the weak interaction doesn't conserve CP either: the main evidence for this comes from the neutral kaon system. (One of us (YK) had precisely this question on our GRE). By the CPT theorem, this means that T by itself must also be violated, and there are various experiments involving the precession of muon spins in a magnetic field which demonstrate this.

2.4 Recent developments

For the Standard Model to be mathematically consistent, it must contain (at least) one particle which has not yet been observed: the *Higgs boson*. This particle is responsible for giving mass to all elementary particles, via a mechanism known as *spontaneous symmetry breaking*. Many measurements have constrained the Higgs mass to lie within a rather narrow range – for this value to be consistent with the principles of quantum field theory, many physicists believe that there must exist a further symmetry of nature known as *supersymmetry*. If this is the case, each elementary particle has a *superpartner* with exactly the same charges, but with spin differing by 1/2. As of this writing, the Large Hadron Collider is hot on the trail of these hypothetical particles, but none have yet been discovered.

On firmer experimental footing is the discovery that *neutrinos have mass*. This was deduced by observing *neutrino oscillations*, a phenomenon by which one flavor of neutrino (say an electron neutrino) is emitted from a source, but later detected as another flavor (say a tau neutrino). Unfortunately, this kind of measurement only permits one to determine mass differences, not absolute masses, but it is known that all neutrinos are extremely light, with masses less than 1/1000 the mass of the electron.

3 Condensed matter physics

4 Astrophysics

4.1 Absorbtion and emission lines; 21cm?

4.2 Redshift, etc

5 Recent Nobel Prizes

The Nobel prizes in physics provide an excellent source for random trivia about current developments in physics. Anecdotally (though we don't have enough data to back up this claim), the GRE likes to throw in questions dealing with recent Nobel prizes, so here is a quick summary.

- **2010: Isolation of graphene.** Two condensed matter physicists isolated graphene by ripping flakes of it off a lump of graphite using sticky tape. Carbon occurs in many different forms in nature (charcoal, diamond, fullerenes, and so forth), known as *allotropes*. Graphite is one of these, and in fact is made up of a huge number of stacked two-dimensional layers known as graphene. In graphene, carbon atoms are arranged in a hexagonal lattice, which results in highly unusual behavior of the covalently bonded electrons: they behave like *massless particles*, with a linear dispersion relation, rather than massive ones which have quadratic dispersion. This makes graphene an excellent conductor, and the fact that it is one atom thick gives rise to many possible engineering applications.

- **2009: Optical fibers and charge-coupled devices.** As you probably know, optical fibers exploit total internal reflection to transmit light over large distances with very little attenuation. The prize was given for a method of fabricating impurity-free glass fibers: this is probably too engineering-heavy to find a place on the GRE.

- **2008: Spontaneously broken symmetry.** One half of this prize was awarded for "the discovery of the mechanism of spontaneous broken symmetry in subatomic physics." To explain how this works in detail would be far beyond what's needed for the GRE, but a few buzzwords might be useful. A system with a certain underlying symmetry can have a ground state which does not respect that symmetry. The standard example is trying to balance a pencil on its tip: even there is no preferred direction for the pencil to fall, it will fall eventually, and pick a direction in doing so. A similar situation can happen in particle physics, and any time the quantum-mechanical ground state does not respect a symmetry which was originally present in the theory, a massless scalar particle appears called a *Nambu-Goldstone boson*. Nambu's original application was to the BCS theory of superconductivity, where gauge invariance is spontaneously broken, giving rise to a massless phonon. But the most well-known application is to the spontaneous breaking of the SU(2) × U(1) gauge symmetry of elementary particle physics, for which the Nambu-Goldstone boson is the Higgs boson. The second half of the prize was awarded for an interesting technical result which implies that CP violation requires at least three families of quarks.

- **2007: Giant magnetoresistance.**

- **2006: CMB anisotropy.** This prize was awarded for discovering that the low-temperature bath of photons pervading the universe, known as the *cosmic microwave background*, has an almost perfect blackbody spectrum, and that the deviations from this spectrum may be hints of structure formation in the early universe. Shortly after the Big Bang, the universe was so hot and dense that photons had an extremely small mean free path. Hence we can't use light to determine anything about the very early universe, because the photons didn't travel in straight lines. After about 380,000 years, though, the universe expanded and cooled enough that the photons hit a *surface of last scattering* after which they were free to stream through the universe unimpeded. This coincided with the *epoch of recombination*, when electrons and protons combined to form hydrogen atoms. At this time, the temperature of the universe was about 3000 K, but since then the universe has expanded so drastically that these photons are now highly *redshifted*. Their temperature today is 2.7 K, corresponding to a blackbody peak at a wavelength of 1.9 mm – this is in the microwave region, hence the name. At a level of about one part in 100,000, the spectrum of this radiation bath deviates from the blackbody spectrum, corresponding to density perturbations in the early universe; many of these have been correlated with the positions of galaxies and galaxy clusters today.